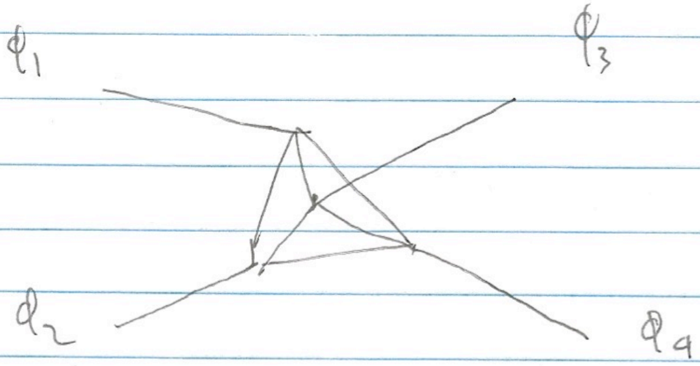
Schwartz
7.7 (a)



$\mathcal{L}_{int} = \frac{\lambda}{4!}\phi^4$ This happens at $O(\lambda^4)$

At $O(\lambda^4)$ $T\{\exp[i\mathcal{L}_{int}(\phi_0)]\}$ looks like

$$\frac{1}{4!}\left(\frac{i\lambda}{4!}\right)^4 T\{\phi_{x_1}^4\ \phi_{x_2}^4\ \phi_{x_3}^4\ \phi_{x_4}^4\}.$$

So we consider wick contractions of

$$\frac{1}{4!}\left(\frac{i\lambda}{4!}\right)^4 \langle 0|T\{\phi_1\phi_2\phi_3\phi_4\ \phi_{x_1}^4\ \phi_{x_2}^4\ \phi_{x_3}^4\ \phi_{x_4}^4\}|0\rangle$$

One possibility is

$$\phi_1\phi_2\phi_3\phi_4\ x_1\ x_1\ x_1\ x_1\ x_2\ x_2\ x_2\ x_2\ x_3\ x_3\ x_3\ x_3\ x_4\ x_4\ x_4\ x_4$$

To take account of all symmetry factors, WLOG, begin with ϕ_1, it must connect to $\{\chi_1, \chi_2, \chi_3, \chi_4\}$ for the diagram to be connected, so it has 16 choices, $(\times 16)$.

WLOG, ϕ_1 connects χ_1, then the χ_1 vertex must connect to neither itself nor any of $\phi_2, \phi_3\ \phi_4$ for the diagram to be connected, it has 12 choices in $\{\phi_2, \phi_3, \phi_4\}$, then 8 choices in $\{\phi_3, \phi_4\}$, then 4 choices in ϕ_4, WLOG, so we pick up factors of $(\times 12 \times 8 \times 4)$.

Removing connections already made, we are left with

$$\phi_2\ \phi_3\ \phi_4 \quad \chi_2\chi_2\chi_2 \quad \chi_3\chi_3\chi_3 \quad \chi_4\chi_4\chi_4.$$

By the same procedure, we pick factors of

$(9 \times 6 \times 3)$ and we are left with

$$\phi_3\ \phi_4 \quad \chi_3\chi_3 \quad \chi_4\chi_4 \Rightarrow (4 \times 2),$$

$$\phi_4 \quad \phi_4 \quad \Rightarrow (1).$$

So the overall symmetry factor is

$$(16 \times 12 \times 8 \times 4) \times (9 \times 6 \times 3) \times (4 \times 2) \times 1$$

$$= (4^5 \times 3 \times 2) \times (3^4 \times 2) \times (2^3) = 4^5 3^5 2^5 = (4!)^5$$

So the $O(\lambda^4)$ term can be computed.

$$\frac{1}{4!}\left(\frac{-i\lambda}{4!}\right)^4 (4!)^5 \int d^4x_1\, d^4x_2\, d^4x_3\, d^4x_4\, D_{1x_1} D_{x_1x_2} \cdots D_{4x_4}$$

$$= \boxed{(-i\lambda)^4 \int (\cdots)}$$

That is, the $\frac{1}{4!}$ cancels the symmetry factors.

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Schwartz
7.7(b)

ϕ^3 theory without external lines

without external lines, we have

$$\langle 0| T\{\phi_{x_1}^3 \phi_{x_2}^3 \phi_{x_3}^3 \phi_{x_4}^3\}|0\rangle$$

We consider connected wick contraction diagrams.

$$x_1x_1x_1 \quad x_2x_2x_2 \quad x_3x_3x_3 \quad x_4x_4x_4$$

$$\Downarrow$$

$$x_1x_1x_1 \quad x_2x_2x_2 \quad x_3x_3x_3 \quad x_4x_4x_4$$

$(9 \times 6 \times 3)$ $\Downarrow$

$$x_1x_1 \quad x_3x_3 \quad x_4x_4$$

$$\Downarrow$$

$$x_1x_1 \quad x_3x_3 \quad x_4x_4$$

(4×2) $\Downarrow$

$$x_1 \, x_4$$

The symmetry factor would have been only

$$(9 \times 6 \times 3) \times (4 \times 2)$$

$$= (3^4 \times 2) \times (2^3)$$

$$= 3^4 \times 2^4 = (3!)^4.$$

In $O(\lambda^4)$ the factor from exponential is

$$\frac{1}{4!}\left(\frac{i\lambda}{3!}\right)^4 \langle 0 | T\{\cdots\} | 0 \rangle$$

so the $\frac{1}{3!}$ in $\frac{\lambda}{3!}$ would still be sufficient to cancel the symmetry factors.

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